Hexagonal Data Sampling

Introduction

The honeycomb is a tessellation of hexagons – an arrangement that does much more than create an attractive pattern. For a given perimeter length, the hexagon provides a greater internal area than tessellated squares or equilateral triangles. For the honeybee, this means more storage space for a given amount of building material. Other notable examples include naturally occurring imaging systems such as the composite eyes of many insects or even the arrangement of retinal sensory elements in the human eye. Interesting, but what is the allure of the hexagonal lattice for Orientation Imaging Microscopy (OIM™)? Since hexagonal grids are generally more difficult to implement than square grid systems, why extend the effort?

Simply noting the differences between the maps in Figure 1 (where the horizontal spacing between data points is identical), one can see how sampling on a hexagonal grid is better suited to characterizing microstructures than sampling on a square grid. This note discusses some of the advantages of hexagonal sampling beyond generating pretty pictures.

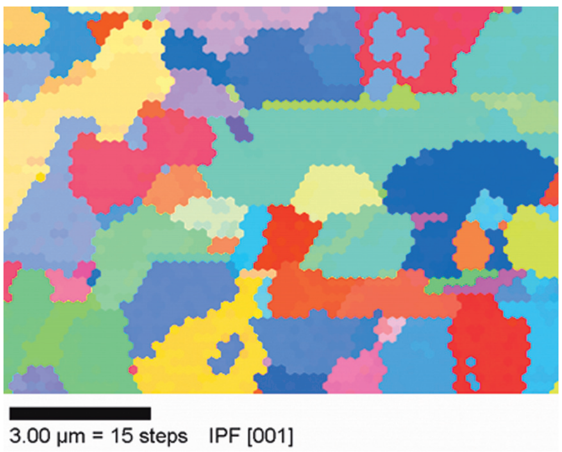

Figure 1a. Hexagonal grid

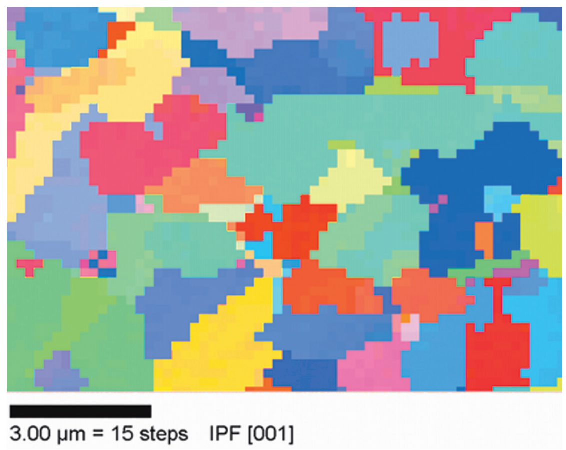

Figure 1b. Square grid

Sampling Efficiency

For axisymmetric features (i.e. a circle), hexagonal sampling is 13% more efficient than square sampling[1,2,3]. Of course, microstructures are not always nice-neat equiaxed structures and the difference in sampling efficiency may not be as dramatic in more elongated structures. Nonetheless, the higher sampling efficiency of the hexagonal grid increases the likelihood of finding and more accurately rendering smaller features.

Ambiguity

In a tessellated hexagonal grid, each and every hexagon (excluding those bordering the edges of the grid) has six equidistant neighbors. Every point can be enclosed within a contiguous series of those six neighbors, as seen in Figure 2. Square arrays introduce an ambiguity in attempting to define a neighborhood[4].

A point may have either four or eight neighbors, depending on whether or not one considers the diagonals in the definition, and neither is optimal. The 4-neighbor definition precludes a contiguous border for a point, whereas the 8-neighbor results in different point-to-point distances. Having six equidistant neighbors offers greater simplicity and accuracy for neighbor connectivity calculations as are used in the grain grouping algorithms and kernel-based plastic strain analysis within OIM™ Analysis.

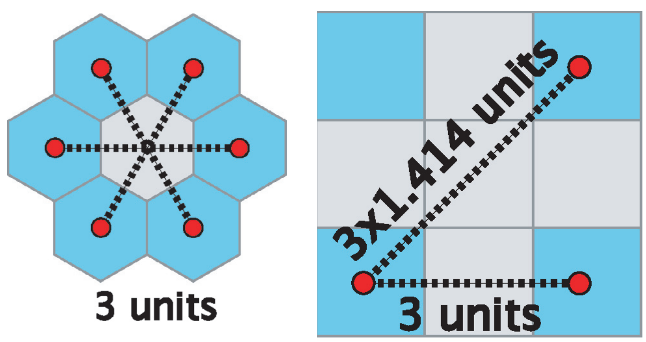

Figure 2.

Boundaries

In OIM™ Analysis, boundaries are found by determining the misorientation between neighboring points on the scanning grid. If the misorientation exceeds some user-defined value, then a line segment is drawn separating the points. The angle of these line segments is limited by the collection grid. For a square grid, the allowable angles are restricted to 0° and 90° and to 0°, 60° and 120° for the hexagonal. This limited range makes it difficult to characterize both the actual angle of a boundary with respect to the sample reference frame as well as the true length of the boundary. Figure 3 shows the errors in boundary line lengths associated with the prescribed grids. The square grid approach has a wider error range across the distribution of possible line angles. A correction factor can be applied to the detected boundary lengths; however, the error for the hexagonal grid would be much smaller (± 0.09) than for a square grid (± 0.2). The hexagonal grid approach also becomes more efficient as the grain shapes become more curved rather than linear.

To alleviate these errors, OIM™ Analysis has the ability to link individual boundary segments together to form "reconstructed boundaries" as shown in Figure 4. The reconstructed boundaries essentially link triple points together; highlighting another inherent advantage of the hexagonal sampling grid – triple points. In three dimensions, the intersection of two boundary planes is a line. In a two dimensional cross-section, these intersection lines appear as triple points. The quadruple points inherent to square grids are simply never observed[5] in real microstructures. The hexagonal grid more closely mimics the polygonal character of real microstructures than square grids.

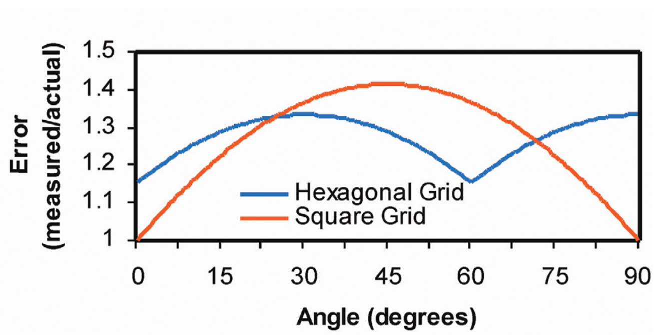

Figure 3. Boundary characterization errors.

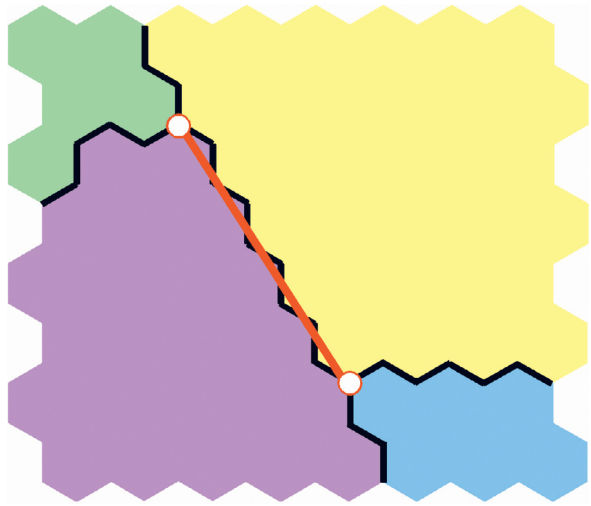

Figure 4. Schematic of a reconstructed boundary.

Conclusion

Hexagonal data provides the optimal spatial and computational characteristics for EBSD data. TEAM™ and OIM™ Analysis have the capability to perform scans and data analysis using both hexagonal grids and square grids. This enables the user to take advantage of the superior sampling capabilities of the hexagonal grid while providing the user the flexibility to use square grids for compatibility with custom post-processing analyses.

Bibliography

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