Elastic Modulus Mapping of Golf Club Heads

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Introduction

In many materials applications, the elastic behavior of a material is of critical importance. The elastic behavior can be described in terms of the elastic modulus. The elastic modulus relates the applied stress to the resulting elastic strain as illustrated in the simple schematic shown in Figure 1.

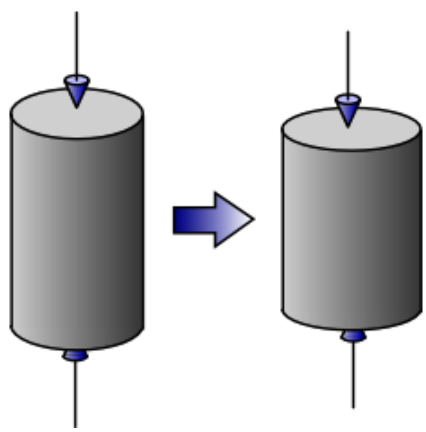

Figure 1 – Schematic showing strain resulting from an applied stress.

If a material exhibits any directionality (or anisotropy) in its elastic behavior, then this anisotropy must to be accounted for in the design process. In fact, the elastic anisotropy may be used advantageously for some advanced applications.

While the elastic anisotropy of single crystals is well known; traditionally, most polycrystalline materials are assumed to be elastically isotropic (that is without any anisotropy). A polycrystal where the orientations of the constituent grains are randomly distributed would exhibit no elastic anisotropy. However, the forming of a polycrystalline material for a particular application tends to cause the constituent crystals to rotate to preferred orientations resulting in a non-random orientation distribution (texture). Since the individual constituent crystals are elastically anisotropic, the textured polycrystal will also exhibit a degree of elastic anisotropy. Thus, if the single crystal elastic anisotropy is known and the texture of a given polycrystal is measured the average anisotropy for the polycrystal can be estimated. Such measurements have been made where textures have been measured using X-Ray diffraction, which provides bulk measurements of texture in polycrystals. While similar measurements can be made with OIM, the real power of OIM is in its ability to examine local variations in orientation. With the proper data manipulations, OIM can provide a unique way to examine local variations in elastic anisotropy.

Mathematical Descriptions of Elastic Anisotropy

While the elastic modulus is simply a scalar value, the direction of strain and direction of stress to relate through the modulus calculation must be specified. The relationship between stress and strain can be described by Hooke's Law [1]. This law is given by a relatively simple equation:

$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl} \tag{1}$$

In this equation σ_{kl} is a tensor describing the applied stress and ε_{ij} is a tensor describing resulting strain. The indices in this expression indicate different directions within the material providing a mathematical description for the anisotropic response of the material. The variable S_{ijkl} in this equation is the fourth rank Elastic Compliance Tensor. This tensor allows the stress and strain in different directions to be related to one another.

The values of the components of the compliance tensor for single crystals (S^{X}_{ijkl}) are tabulated in reference books on materials properties [2]. It is important to note that the tensors defined in equation (1) are defined relative to the sample reference system; the single crystal compliance tensor is defined relative to the principle axes of the crystal lattice. At each point in the OIM scan the orientation of the crystal lattice with respect to the sample reference system is determined. If we express the orientation of the crystal lattice as a matrix as g_{ij} then the compliance tensor for a given grain of orientation g_{ij} can be expressed as follows:

$$S'_{ijkl} = g^{T}_{ip}g^{T}_{ip}g^{T}_{ip}g^{T}_{ip}S^{X}_{pqrs} \tag{2}$$

where S^{X}_{pqrs} is the tabulated single crystal compliance tensor and S'_{ijkl} is the rotated compliance tensor. T denotes the transpose of the matrix.

The compliance tensor for the bulk material can be estimated from an OIM scan containing n orientation measurements using the following equations (termed the Reuss and Voigt averages – C_{ijkl} is the stiffness tensor which is the inverse of the compliance tensor: $\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$):

$$\overline{S}_{ijkl} = \frac{1}{N}\sum_{n=1}^{N} S'^{n}_{pqrs} \quad \text{and} \quad \overline{C}_{ijkl} = \frac{1}{N}\sum_{n=1}^{N} C'^{n}_{pqrs} \tag{3}$$

Thus, OIM facilitates the calculation of two important parameters: 1) the tensor values at individual points in the microstructure and 2) an estimate of the average compliance tensor. Since the compliance tensor is a fourth rank tensor containing 81 values (some of these are not unique because of crystal symmetry – i.e. for cubic crystals the tensor contains only 3 unique values and for hexagonal crystal symmetry five unique values) it is difficult to generate a meaningful map of the tensor values at individual points in the OIM scan. It is more practical to generate a map based on a given elastic modulus. The modulus can be calculated for a given stress and strain. For example, the modulus for the stress and strain in the sample normal direction is given by $\sigma_{33}/\varepsilon_{33}$. This value can be mapped onto a color scale to create color maps showing the variation in elastic modulus for any specified direction.

Example: Golf Club Heads

The map in Figure 2 shows the elastic modulus for the strain in a direction normal to the page resulting from uniaxial compression in the same direction. The material is a sample cored from a steel iron. In this map, the points colored in red are stiffer than the points in blue.

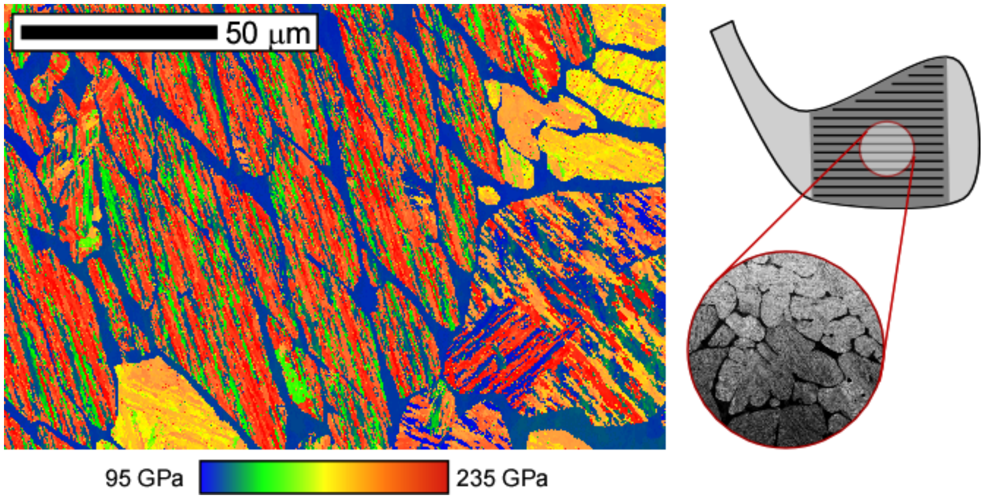

Figure 3 – An elastic modulus map for a forged steel golf club head. The modulus mapped is that for uniaxial compression in a direction normal to the club face..

Figure 3 shows maps for image quality, an inverse pole figure map and a modulus map for a sample cored from the face of a cast titanium golf club head. Note the complex Widmanstätten structure of the titanium in this microstructure.

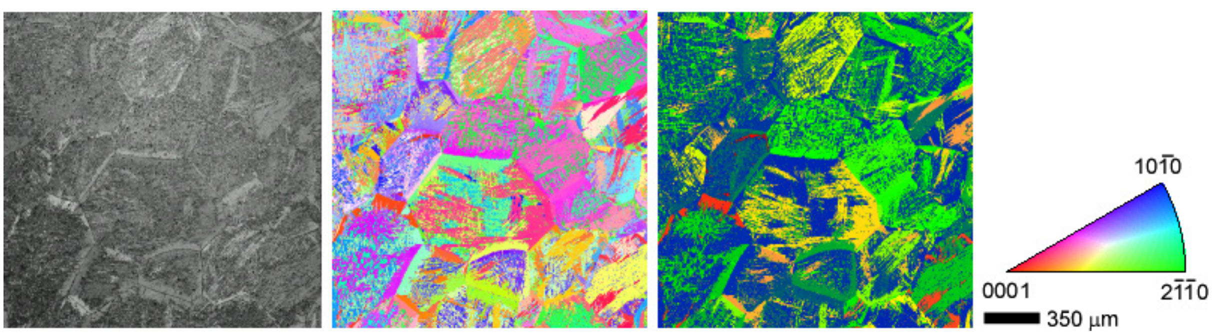

Figure 3 – Image Quality, Inverse Pole Figure and Elastic modulus maps for a cast titanium driver sample. The color streaographic triangle at right gives the coloring scheme for the inverse pole figure map – e.g. points with c-axes normal to the sample surface are colored red.

Figure 4 shows the distribution of elastic moduli for two different cast titanium drivers, a forged titanium driver, a forged steel iron and a cast steel iron.

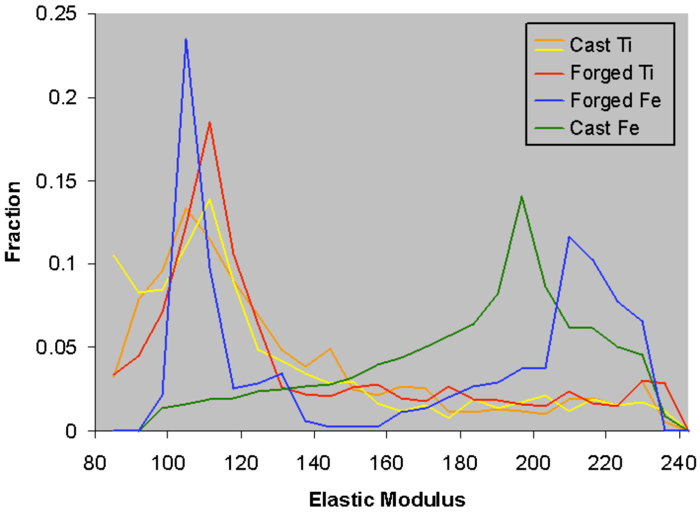

Figure 4 – Distribution of elastic moduli for various golf club head materials.

Conclusions

The local variability in elastic modulus can be quite critical for golf club performance. Especially if the grain size in the material is fairly coarse. For example, consider a club where the grain size is on the same scale as the impact zone of the ball on the club face. Then make the simplistic assumption that driving distance is solely dependent on the stiffness of the material in the ball impact zone. For the case of titanium the maximum stiffness is three times the minimum stiffness. Thus, if two neighboring grains were oriented such that one grain was in the maximum stiffness orientation and the other was in the minimum stiffness orientation, then two balls struck in only slightly different positions on the club face would result in one ball being driven three times farther than the other.

Of course, in the end, even a club with optimized microtexture is not going to fix a bad swing!

Bibliography

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[2] K.-H. Landwege and A. M. Helwege, Eds. (1979). *Landolt-Börnstein Numerical Data and Functional Relationships in Science and Technology*, vol. III/11, Springer Verlag: Berlin.

For more details on this work see the following reference.

M. M. Nowell and S. I. Wright. (2001). Microtextural Characterization of Golf Club Heads. *Materials Science in Sports*, F. H. Froes, & S. J. Haake, (Eds.), pp. 119-131. TMS: Warrendale, PA.